

## Thermal & Fluids PE Exam Technical Study Guide Errata

This product has been updated to incorporate all changes shown in the comments on the webpage and email comments as of January, 01 2020. If you have purchased this product prior to this date and wish for the latest version then please email Justin Kauwale at [contact@engproguides.com](mailto:contact@engproguides.com).

The second step in finding the convective heat transfer coefficient is to find the Prandtl number. This number describes the fluid and its ability to store heat and transfer heat through conduction.

$$Pr = \frac{\mu * c_p}{k}$$

$$c_p = \text{specific heat} \left( \frac{Btu}{lbm * ^\circ F} \right); k = \text{thermal conductivity} \left( \frac{Btu}{h * ft^2 * \frac{^\circ F}{ft}} \right);$$

$$\mu = \text{dynamic viscosity} \left( \frac{lbm}{ft - hr} \right)$$

The Prandtl numbers for typical fluids are as follows:

Fluid	Pr
Air	0.7
Water	7
Seawater (32 F)	13.4
Seawater (68 F)	7.2

The final step is the most difficult step and it is to use the Nusselt number using the Reynolds and Prandtl number. Once solved, the following equations can be used to solve for the convective heat transfer coefficient.

$$Nu = \frac{D * h_{conv}}{k} = C$$

where  $D$  = diameter of pipe (ft),  $k$  = thermal conductivity of the fluid

The value to the right of the equation is a constant that is determined from one of the following equations depending on the scenario you encounter.

#### 4.2.1 Turbulent flow inside circular pipe (cooling/heating)

$$Nu = \frac{D * h_{conv}}{k} = .023 * Re^{.8} * Pr^{\frac{1}{3}} \left( \frac{\mu_{bulk}}{\mu_{surface}} \right)^{.14}$$

$$\text{when } Re \geq 10,000; 0.7 \leq Pr \leq 16,700, \text{ and } \frac{L}{D} \geq 60$$

where  $\mu_{bulk}$  = fluid viscosity at bulk(avg) fluid temp;  $\mu_{surface}$   
= fluid viscosity at inner pipe surface temp

The above equation is the Sieder and Tate estimation of the Nusselt Number for typical fluids in long straight pipe, under turbulent flow. The equation takes into account the fluid viscosity



changes due to the temperature differences between the inner pipe surface and the bulk fluid. The above conditions for the Reynolds and Prandtl numbers must apply.

#### 4.2.3 Laminar flow inside circular pipe (heating)

$$Nu = \frac{D * h_{conv}}{k} = 4.36$$

This equation works when it is assumed that there is uniform heat transfer through the pipe and laminar flow. The Reynolds number must be less than 2300 to qualify as laminar flow.

#### 4.2.3 Laminar flow inside circular pipe (cooling)

$$Nu = \frac{D * h_{conv}}{k} = 4.66$$

This equation works when it is assumed that the surface temperature of the pipe is constant and there is laminar flow. The Reynolds number must be less than 2300 to qualify as laminar flow.

#### 4.2.4 Other situations

There are many other types of situations, each with their own Nusselt equation. You do not need to memorize these specific situations but you should be aware of these equations in your references. These equations can be found in your Mechanical Engineering Reference Manual under the Heat Transfer section.

Once you have the convective heat transfer coefficient then you can use the below equation to calculate the convective heat transfer.

$$Q = h_{conv} * A * \Delta T$$

$$h_{conv} = \text{convective heat transfer coefficient} \left[ \frac{Btu}{hr * ft^2 * ^\circ F} \right]$$

$$A = \text{area of heat transfer} [ft^2]$$

$$\Delta T = \text{temperature difference between hot and cold areas of heat transfer} [^\circ F]$$

## 5.0 RADIATION

*Radiation is the mode of heat transfer that requires no substance to transmit heat.*

### 8.3 PROBLEM 3: REYNOLDS NUMBER

250 gpm of 120°F water flows through a 4" diameter pipe that is located in the ceiling of a building. Assume the water in the pipe is not under extreme pressure. Calculate the Reynolds number of the fluid inside the pipe.

- a) 57,734
- b) 295,941
- c) 355,745
- d) 136,836,344

### 8.4 PROBLEM 4: CONVECTIVE HEAT TRANSFER COEFFICIENT

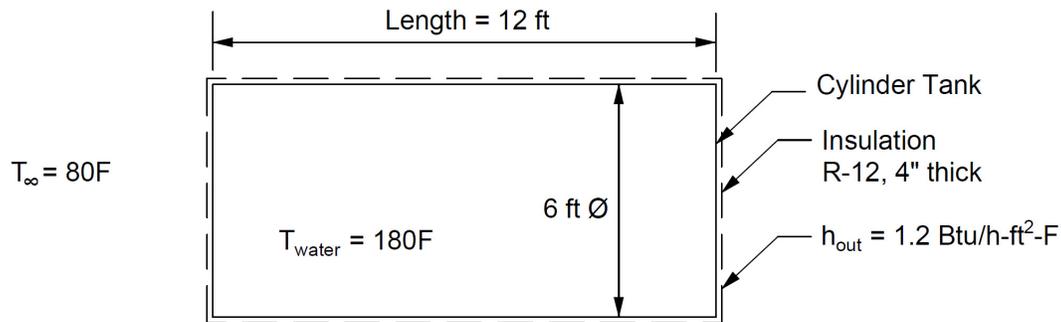
250 gpm of 120°F water flows through a 4" diameter pipe that is located in the ceiling of a building. Calculate the convective heat transfer at the fluid inside the pipe. Assume the water in the pipe is not under extreme pressure.

Heat capacity of water is 1.0 Btu/lbm-F. Dynamic viscosity of water is 1.36 lbm/ft-hr. Assume no change in dynamic viscosity of water as a function of temperature. Conductivity of water is 0.378 Btu-h/ft<sup>2</sup>-F

- a)  $1,067 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$
- b)  $3,182 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$
- c)  $6,738 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$
- d)  $12,784 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}}$

## 8.5 PROBLEM 5: CONDUCTION

A horizontal, cylindrical tank holds water at 180F. The tank is 12 ft long, has a 6 ft diameter, and is insulated by 4" thick, R-12 insulation. The tank is located in a room at 80F and is raised off of the ground. The tank ends are flat, assume that the heat transfer through the tank wall is negligible, and that the exterior surface coefficient is 1.2 Btu/hr ft<sup>2</sup> F. The tank is completely filled. What is the rate of heat transfer to the space? Assume no internal convective heat transfer.



- (a)  $2300 \frac{\text{Btu}}{\text{h}}$
- (b)  $3600 \frac{\text{Btu}}{\text{h}}$
- (c)  $3900 \frac{\text{Btu}}{\text{h}}$
- (d)  $4200 \frac{\text{Btu}}{\text{h}}$

## 8.6 PROBLEM 6: CONVECTION

Which term is not applicable to calculating the convection coefficient of a forced convection fluid, regardless of the fluid viscosity and turbulence?

- (a) Nusselt Number
- (b) Reynold's Number
- (c) Grashof Number
- (d) Prandtl Number

## 8.7 PROBLEM 7: PIPE RESISTANCES

A pipe has an outer diameter of 1.315 inches and an inner diameter of 1.049 inches. The thermal conductivity of the pipe material is 20 Btu/h-ft-°F. The inner convective heat transfer coefficient between the fluid within the pipe and the inner pipe walls is 150 Btu/h-ft<sup>2</sup>-°F. The outer convective heat transfer coefficient between the outer pipe wall and ambient air is 50 Btu/h-ft<sup>2</sup>-°F. **What is the overall heat transfer coefficient per foot of pipe?**

- (a) 12 Btu/hr-ft-°F
- (b) 42 Btu/hr-ft-°F
- (c) 85 Btu/hr-ft-°F
- (d) 107 Btu/hr-ft-°F

## 8.8 PROBLEM 8: PIPE RESISTANCES

A pipe has an outer diameter of 4.2 inches and an inner diameter of 4.0 inches. The thermal conductivity of the pipe material is 10 Btu/h-ft-°F. The inner convective heat transfer coefficient between the fluid within the pipe and the inner pipe walls is 35 Btu/h-ft<sup>2</sup>-°F. The outer convective heat transfer coefficient between the outside insulation layer and ambient air is 5 Btu/h-ft<sup>2</sup>-°F. The pipe has 2 inches of insulation. The insulation has a thermal conductivity of 0.05 Btu/h-ft-°F. If the temperature of the fluid within the pipe is 300 °F and the ambient air outside of the pipe is 75 °F, then what is the rate of heat transfer per foot of pipe?

- (a) 105 Btu/hr
- (b) 671 Btu/hr
- (c) 1,743 Btu/hr
- (d) 2,990 Btu/hr

$$v = \frac{\dot{V}}{A} = \frac{0.566 \frac{ft^3}{sec}}{\pi * \left(\frac{0.33 ft}{2}\right)^2} = 6.5 \frac{ft}{s}, \quad \text{velocity}$$

Use the tables in the NCEES Mechanical PE Reference Handbook to find the kinematic viscosity of water at 120°F. Because the properties of water do not change drastically under minor pressure differences, water at atmospheric pressure can be used as a close estimate.

$$\nu = 0.609 \times 10^{-5} \frac{ft^2}{s}, \quad \text{kinematic viscosity}$$

Solve for Reynolds number:

$$Re = \frac{D * v}{\nu} = \frac{(0.33 ft) * 6.5 \frac{ft}{s}}{0.609 \times 10^{-5} \frac{ft^2}{s}} = 355,775$$

The correct answer is most nearly, (C) 355,745

## 9.4 SOLUTION 4: CONVECTIVE HEAT TRANSFER COEFFICIENT

This is a forced convective heat transfer problem through a pipe and the objective is to solve for the convective heat transfer,  $h_{conv}$ . Use the Reynolds number in Problem 3 to determine whether the flow is laminar or turbulent. Because the Reynolds number is greater than 2,300, the flow is turbulent

The pipe is within the ceiling, exposed to ambient temperatures, meaning the temperature of the pipe wall temperature is lower than the fluid temperature. You need equation 5.3.9 for the turbulent flow in circular tubes to solve for the convective heat transfer coefficient. Please see pages 280 and 281.

Therefore, Equation 4.1.2 can be used to solve for the convective heat transfer coefficient,  $h_{conv}$ .

$$Nu = \frac{D * h_{conv}}{k} = .023 * Re^{.8} * Pr^{.333} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$\frac{\mu_b}{\mu_s} = 1; \text{ There is no change in the dynamic viscosity.}$$

Solve for the Nusselt number, Nu, by first finding the Prandtl number. The Prandtl number, Pr, is calculated with the following equations.

$$Pr = \frac{\mu * c_p}{k}$$

Use the tables in the **NCEES Mechanical PE Reference Handbook** to find the properties of water. Because these properties of water do not change drastically under minor pressure differences, water at atmospheric pressure can be used as a close estimate.

At 120°F, water has the following properties:

$$\mu = 1.36 \frac{\text{lbm}}{\text{ft-hr}}, \text{dynamic viscosity}$$

$$c_p = 1.0 \frac{\text{Btu}}{\text{lbm} * ^\circ\text{F}}, \text{specific heat}$$

$$k = 0.378 \frac{\text{Btu} * \text{ft}}{\text{h} * \text{ft}^2 * ^\circ\text{F}}, \text{thermal conductivity (given in problem)}$$

Therefore,

$$Pr = \frac{\mu * c_p}{k} = \frac{1.36 \frac{\text{lbm} * \text{ft}}{\text{hr}} * 1.0 \frac{\text{Btu}}{\text{lbm} * ^\circ\text{F}}}{0.378 \frac{\text{Btu} * \text{ft}}{\text{h} * \text{ft}^2 * ^\circ\text{F}}} = 3.6$$

Solve for the convective heat transfer:

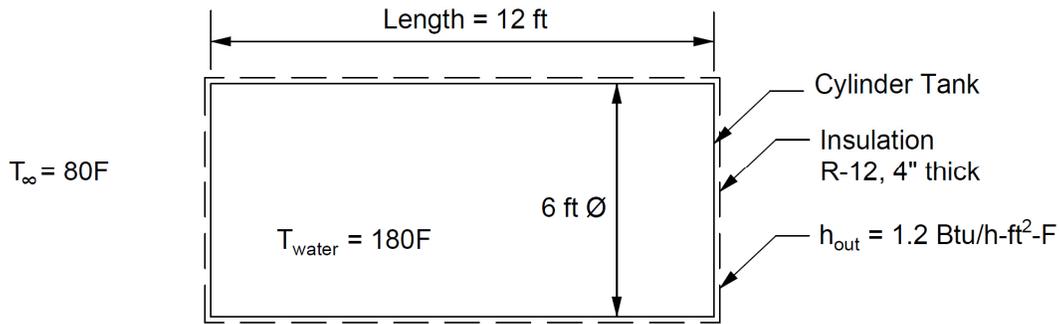
$$h_{conv} = \frac{.023 * Re^{.8} * Pr^{.3} * k}{D} = \frac{.023 * 355,745^{.8} * 3.6^{.3} * 0.378 \frac{\text{Btu} * \text{ft}}{\text{h} * \text{ft}^2 * ^\circ\text{F}}}{0.33 \text{ ft}}$$

$$h_{conv} = 6,738 \frac{\text{Btu}}{\text{h} * \text{ft}^2 * ^\circ\text{F}}$$

The correct answer is most nearly, (A) 1,067  $\frac{\text{Btu}}{\text{h} * \text{ft}^2 * ^\circ\text{F}}$

## 9.5 SOLUTION 5: CONDUCTION

A horizontal, cylindrical tank holds water at 180F. The tank is 12 ft long, has a 6 ft diameter, and is insulated by 4" thick, R-12 insulation. The tank is located in a room at 80F and is raised off of the ground. The tank ends are flat, assume that the heat transfer through the tank wall is negligible, and that the exterior surface coefficient is 1.2 Btu/hr ft<sup>2</sup> F. The tank is completely filled. What is the rate of heat transfer to the space? Assume no internal convective heat transfer.



(a)  $2300 \frac{\text{Btu}}{\text{h}}$

(b)  $3600 \frac{\text{Btu}}{\text{h}}$

(c)  $3900 \frac{\text{Btu}}{\text{h}}$

(d)  $4200 \frac{\text{Btu}}{\text{h}}$

Find the heat transfer rates through both ends and the cylinder. For a flat plate, use the following equation.

$$Q_{flat\ ends} = UA \Delta T$$

The U value will account for the tank insulation and the convection in series.

$$U_{flat\ ends} = \frac{1}{R_{ins} + \frac{1}{h_{outer}}} = \left( \frac{1}{12 \frac{\text{h} \cdot \text{ft}^2 \cdot \text{F}}{\text{Btu}} + \frac{1}{1.2 \text{ Btu}/(\text{h} \cdot \text{ft}^2 \cdot \text{F})}} \right) = 0.078 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot \text{F}}$$

$$Q_{flat\ ends} = 0.078 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2 \cdot \text{F}} * \left[ 2 * \left( \pi \frac{(6 \text{ ft})^2}{4} \right) \right] * (180\text{F} - 80\text{F}) = 440.6 \frac{\text{Btu}}{\text{h}}$$

The heat transfer rate through a cylinder wall is found with the following equation, accounting for the tank insulation and the outer convection in series. In this problem the tank wall is neglected and the inner convection coefficient is assumed to be zero.

$$Q_{cylinder} = \frac{2\pi L(\Delta T)}{\frac{\ln\left(\frac{r_{outer}}{r_{inner}}\right)}{k_{ins}} + \frac{1}{r_{outer}h_{outer}}}$$

Find the tank insulation in terms of convection coefficient, k.

$$k_{ins} = \frac{t_{ins}}{R_{ins}} = \frac{4\text{in} * \frac{1\text{ft}}{12\text{in}}}{12 \frac{\text{h} \cdot \text{ft}^2 \cdot \text{F}}{\text{Btu}}} = 0.028 \frac{\text{Btu}}{\text{h} \cdot \text{ft} \cdot \text{F}}$$

Note that the outer surface heat transfer coefficient = radiative heat transfer coefficient + outer convective heat transfer coefficient; the radiative and convective heat transfer at the outer surface are in parallel. This combined value is given,  $h_{outer} = h_{conv,outer} + h_{rad} = 1.2 \frac{Btu}{h \cdot ft^2 \cdot F}$

$$Q_{cylinder} = \frac{2\pi * 12ft * (180F - 80F)}{\ln\left(\frac{3ft + \frac{4}{12}ft}{3ft}\right) + \frac{1}{0.028 \frac{Btu}{h \cdot ft \cdot F} + \left(3ft + \frac{4}{12}ft\right) * 1.2 \frac{Btu}{h \cdot ft^2 \cdot F}}} = 1878.9 \frac{Btu}{h}$$

Finally, solve for the total heat transfer between then 180F fluid and the 80F room.

$$Q_{total} = Q_{flat\ end} + Q_{cylinder} = 440.6 \frac{Btu}{h} + 1878.9 \frac{Btu}{h} = 2319.5 \frac{Btu}{h}$$

The answer is most nearly **(a) 2300  $\frac{Btu}{h}$** .

## 9.6 SOLUTION 6: CONVECTION

Which term is not applicable to calculating the convection coefficient of a forced convection fluid, regardless of the fluid viscosity and turbulence?

- (a) Nusselt Number
- (b) Reynold's Number
- (c) Grashof Number
- (d) Prandtl Number

The correct answer is **(c) Grashof Number**. The Grashof number is the ratio of buoyancy forces to viscous forces and is used to calculate the convection coefficient for heat transfer from natural convection. Natural convection is where the fluid is moving due to temperature differences, like the temperature difference between a surface and the ambient air. Forced convection is due to the forced movement of the fluid, like water flowing in a pipe. The Nusselt Number, Reynold's Number, and Prandtl Number are all used to calculate forced convection coefficients. Prandtl and Nusselt numbers are also used in natural convection calculations.

## 9.7 SOLUTION 7: PIPE RESISTANCES

A pipe has an outer diameter of 1.315 inches and an inner diameter of 1.049 inches. The thermal conductivity of the pipe material is 20 Btu/h-ft-°F. The inner convective heat transfer coefficient between the fluid within the pipe and the inner pipe walls is 150 Btu/h-ft<sup>2</sup>-°F. The outer convective heat transfer coefficient between the outer pipe wall and ambient air is 50 Btu/h-ft<sup>2</sup>-°F. **What is the overall heat transfer coefficient per length of pipe?**

The heat transfer area changes with the pipe radius. Therefore, the area is kept integral to each layer of pipe and the composite heat transfer coefficient is taken over a generic area, U\*A. The question is asking for the heat transfer coefficient per length of pipe, so solve for U\*A/L.

First, convert all heat transfer coefficients to resistances.

$$R_{inner,convective} = \frac{1}{150} = 0.0067 \left( \frac{h-ft^2-^{\circ}F}{Btu} \right)$$

$$R_{outer,convective} = \frac{1}{50} = 0.02 \left( \frac{h-ft^2-^{\circ}F}{Btu} \right)$$

For the conductivity through the pipe, you need the equivalent thickness of a pipe.

$$t_{equiv} = r_2 \ln \left( \frac{r_2}{r_1} \right) = \left( \frac{1.315 \text{ in}}{2} \right) \ln \left( \frac{\frac{1.315 \text{ in}}{2}}{\frac{1.049 \text{ in}}{2}} \right) = 0.1486 \text{ in} = 0.01238 \text{ ft}$$

$$R_{pipe,conductive} = \frac{0.01238 \text{ ft}}{20 \frac{Btu}{h-ft-^{\circ}F}} = 0.000619 \frac{h-ft^2-^{\circ}F}{Btu}$$

Finally, add up all the resistances, since all the materials are in series. You need to be sure to multiply the resistance by the inverse of the applicable area per unit length of pipe.

$$\text{Inner Area per ft of pipe} = 2\pi r_{inner}L = 2\pi * (0.5245 \text{ in}) \left( \frac{1}{12} \right) (1 \text{ ft}) = 0.275 \text{ ft}^2/\text{ft}$$

$$\text{Outer Area per ft of pipe} = 2\pi r_{outer}L = 2\pi * (0.6575 \text{ in}) \left( \frac{1}{12} \right) (1 \text{ ft}) = 0.344 \text{ ft}^2/\text{ft}$$

$$R_{equivalent} * \frac{ft}{A} = \left( 0.0067 \frac{h-ft^2-^{\circ}F}{Btu} \right) \left( \frac{1 \text{ ft}}{0.275 \text{ ft}^2} \right) + \left( 0.02 \frac{h-ft^2-^{\circ}F}{Btu} \right) \left( \frac{1 \text{ ft}}{0.344 \text{ ft}^2} \right) + \left( 0.000619 \frac{h-ft^2-^{\circ}F}{Btu} \right) \left( \frac{1 \text{ ft}}{0.344 \text{ ft}^2} \right) = 0.0843 \frac{h-^{\circ}F-ft}{Btu}$$

$$\frac{U_{equivalent} * A}{ft} = \frac{A}{R_{equivalent} * ft} = 11.9 \frac{Btu}{h-ft-^{\circ}F}$$

The correct answer is most nearly, **(a) 12 Btu/hr-ft-°F.**



- (a) 12 Btu/hr-ft-°F
- (b) 42 Btu/hr-ft-°F
- (c) 85 Btu/hr-ft-°F
- (d) 107 Btu/hr-ft-°F

Alternatively, you may directly solve for this problem using the heat transfer for pipe equation. Although this equation is not directly given in the *NCEES Mechanical PE Reference Handbook*, you should be comfortable with how it relates to what is available in the cylindrical wall topic or the heat exchanger section of the Heat Transfer chapter.

$$\frac{UA}{L} = \frac{Q_{cond+conv}}{L * (T_{fluid} - T_{ambient})} = \frac{2\pi}{\frac{\ln\left(\frac{r_2}{r_{inner}}\right)}{k_i} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_{ii}} + \dots + \frac{\ln\left(\frac{r_{outer}}{r_n}\right)}{k_m} + \frac{1}{r_{inner}h_{inner}} + \frac{1}{r_{outer}h_{outer}}}$$

## 9.8 SOLUTION 8: PIPE RESISTANCES

A pipe has an outer diameter of 4.2 inches and an inner diameter of 4.0 inches. The thermal conductivity of the pipe material is 10 Btu/h-ft-°F. The inner convective heat transfer coefficient between the fluid within the pipe and the inner pipe walls is 35 Btu/h-ft<sup>2</sup>-°F. The outer convective heat transfer coefficient between the outside insulation layer and ambient air is 5 Btu/h-ft<sup>2</sup>-°F. The pipe has 2 inches of insulation. The insulation has a thermal conductivity of 0.05 Btu/h-ft-°F. If the temperature of the fluid within the pipe is 300 °F and the ambient air outside of the pipe is 75 °F, then what is the rate of heat transfer per foot of pipe?

First, convert all heat transfer coefficients to resistances.

$$R_{inner,convective} = \frac{1}{35} = 0.029 \left( \frac{h-ft^2-°F}{Btu} \right)$$

$$R_{outer,convective} = \frac{1}{5} = 0.20 \left( \frac{h-ft^2-°F}{Btu} \right)$$

For the conductivity through the pipe, you need the equivalent thickness of a pipe.

$$t_{equiv} = r_2 \ln\left(\frac{r_2}{r_1}\right) = \left(\frac{4.2 in}{2}\right) \ln\left(\frac{\frac{4.2 in}{2}}{\frac{4 in}{2}}\right) = 0.1025 in = 0.00853 ft$$

$$R_{pipe,conductive} = \frac{0.00853 ft}{10 \frac{Btu}{h-ft-°F}} = 0.000853 \frac{h-ft^2-°F}{Btu}$$

For the conductivity of the insulation, you need the equivalent thickness of the insulation.

$$t_{equiv} = r_2 \ln\left(\frac{r_2}{r_1}\right) = \left(\frac{8.2 \text{ in}}{2}\right) \ln\left(\frac{\frac{8.2 \text{ in}}{2}}{\frac{4.2 \text{ in}}{2}}\right) = 2.743 \text{ in} = 0.2286 \text{ ft}$$

$$R_{insulation,conductive} = \frac{0.2286 \text{ ft}}{0.05 \frac{\text{Btu}}{\text{h-ft}^\circ\text{F}}} = 4.572 \frac{\text{h-ft}^\circ\text{F}}{\text{Btu}}$$

Next, add up all the resistances, since all the materials are in series. You need to be sure to multiply the resistance by the inverse of the applicable area.

$$\text{Inner Area} = 2\pi r_{inner}L = 2\pi * (2 \text{ in}) \left(\frac{1}{12}\right) (1 \text{ ft}) = 1.0472 \text{ ft}^2$$

$$\text{Outer Area} = 2\pi r_{outer}L = 2\pi * (2.1 \text{ in}) \left(\frac{1}{12}\right) (1 \text{ ft}) = 1.0996 \text{ ft}^2$$

$$\text{Insulation Outer Area} = 2\pi r_{outer,insulation}L = 2\pi * (4.1 \text{ in}) \left(\frac{1}{12}\right) (1 \text{ ft}) = 2.1468 \text{ ft}^2$$

$$\begin{aligned} R_{equivalent} &= \left(0.029 \frac{\text{h-ft}^\circ\text{F}}{\text{Btu}}\right) \left(\frac{1}{1.0472 \text{ ft}^2}\right) + \left(0.02 \frac{\text{h-ft}^\circ\text{F}}{\text{Btu}}\right) \left(\frac{1}{2.1468 \text{ ft}^2}\right) \\ &+ \left(0.000853 \frac{\text{h-ft}^\circ\text{F}}{\text{Btu}}\right) \left(\frac{1}{1.0996 \text{ ft}^2}\right) + \left(4.572 \frac{\text{h-ft}^\circ\text{F}}{\text{Btu}}\right) \left(\frac{1}{2.1468 \text{ ft}^2}\right) \\ &= 2.167 \frac{\text{h}^\circ\text{F}}{\text{Btu}} \end{aligned}$$

$$U_{equivalent} = \frac{1}{R_{equivalent}} = 0.461 \frac{\text{Btu}}{\text{h}^\circ\text{F}}$$

Finally, multiply by the driving force, which is the difference in temperature.

$$Q = 0.461 \frac{\text{Btu}}{\text{h}^\circ\text{F}} * (300 - 75) = 105.075 \frac{\text{Btu}}{\text{h}} \text{ per linear foot}$$

**The correct answer is most nearly, (a) 105 Btu/hr.**

- (a) 105 Btu/hr
- (b) 671 Btu/hr
- (c) 1,743 Btu/hr
- (d) 2,990 Btu/hr



**Brake Horsepower (BHP):** Brake horsepower is the measure of the power drawn by the motor to turn the fan. BHP is a function of the fan efficiency and the mechanical horsepower.

$$BHP = MHP * \left( \frac{1}{fan\ efficiency} \right)$$

**Horsepower (HP):** Horsepower is the size of the motor. Motors come in standard sizes. [1, 1.5, 2, 3, 5, 7.5, 10, 15, 20, 25, 30, 40, 50, etc] Horsepower is calculated through the following equation and then rounded up to nearest motor size. In the P.E. exam, if the question explicitly asks for the motor horsepower in standard size then calculate the motor horsepower through the below equation and then round up to the nearest motor size. If the question does not ask for standard motor size, then simply provide the output of the below equation.

$$HP = BHP * \left( \frac{1}{motor\ efficiency} \right)$$

*\*upsized HP to nearest motor size*

**Velocity Pressure (VP):** Velocity pressure is defined as the pressure caused solely by moving air.

$$VP = \left( \frac{FPM}{4005} \right)^2 [in. wg] \text{ for standard air density } \rightarrow \rho_{air} = 0.075 \frac{lbm}{ft^3};$$

$$VP = \rho_{air} \left( \frac{FPM}{1097} \right)^2 [in. wg] \text{ for any air density}$$

**Static Pressure (SP):** Static pressure is the pressure caused solely by compression, the outward force on a duct.

**Total Pressure (TSP):** Total static pressure is the sum of the velocity pressure and the static pressure at any point.

## 5.2 TYPES OF FANS

### 5.2.1 AXIAL FANS

Axial fans consist of a fan shaft with fan blades attached around the shaft. Air travels along the axis of the fan and is blown out. These fans are not as common in the residential and commercial HVAC & Refrigeration fields and are more common in industrial ventilation type situations. Within the family of axial fans there are also different types of fans, like the propeller, tube axial and vane axial fans.

### 5.2.2 PROPELLER FANS

Propeller type axial fans consist of a propeller fan in fan housing. This fan, similar to all axial type fans is only suitable for lower pressures.



## 11.18 PROBLEM 18 – DISTRIBUTION SYSTEMS

Water is pumped from tank 1 to tank 2 in the figure below at a rate of 100 GPM. Both tanks are open to atmosphere. The system is made up of 20 ft of 3" schedule 40, welded steel piping with flanged fittings. Assume a roughness coefficient of 0.0018 inches, four 90° long radius elbows, two gate valves, one sharp entrance, and one sharp exit. What will be the pressure required at the pump?

Assume the following conditions:

Viscosity of water  $\nu = 1.08 \times 10^{-5} \text{ ft}^2/\text{s}$

Schedule 40 Steel Pipe Inside Diameter = 3.07 inches

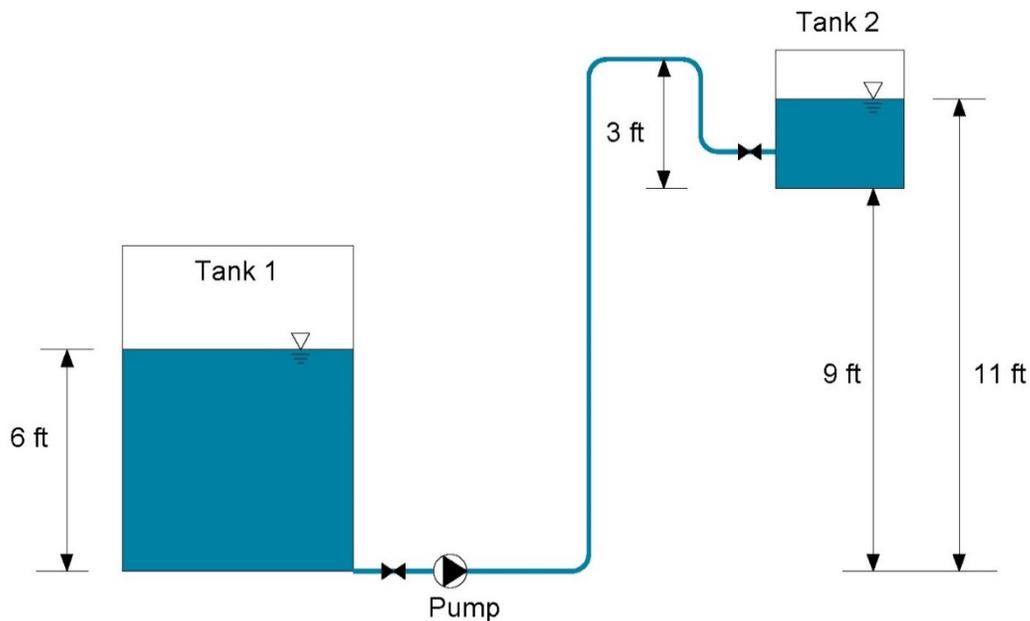
Schedule 40 Steel Pipe Outside Diameter = 3.5 inches

Sharp Exit,  $K=1.0$

Sharp Entrance,  $K=0.5$

3" Flanged 90° long radius elbows,  $K=0.25$

3" Flanged 90° gate valve,  $K=0.22$



- (A) 5.0 ft
- (B) 6.3 ft
- (C) 7.3 ft
- (D) 8.4 ft



reservoir is located 150 feet above the first reservoir. Assume that the friction loss in the piping is 4 psi. What is the minimum motor horsepower required of the pump. Assume the pump has an efficiency of 69% and the motor has an efficiency of 90%.

Refer to your NCEES Mechanical PE Reference Handbook for the hydraulic horsepower equations

$$\text{Mechanical work} = \frac{\Delta P * Q}{1714}$$

$\Delta P$  = pressure drop in psi;  $Q$  = flow rate in GPM

We need to convert 150 feet of head to PSI

$$(\text{ft of head}) = (\text{psi}) * \frac{2.31}{SG}$$

$$(150 \text{ ft of head}) = (P \text{ psi}) * \frac{2.31}{1.75}$$

$$114 \text{ psi} = P_{\text{elevation}}$$

$$\text{Mechanical work} = \frac{(P_{\text{elevation}} + P_{\text{friction}}) * Q}{1714}$$

$$\text{Mechanical work} = \frac{(114 + 4) * 50}{1714} = 3.44 \text{ MHP}$$

$$\text{Shaft work} = \frac{\text{Mechanical work}}{\text{pump efficiency}} = \frac{3.44}{.69} = 4.99 \text{ BHP}$$

Motors are rated based on the output work at the shaft. Thus the motor would need to be at least 4.99 HP, so the next largest motor would be 5 HP.

$$\text{Input motor horsepower} = \frac{\text{Pump work}}{\text{motor efficiency}} = \frac{4.99}{.90} = 5.54 \text{ HP}$$

The input motor horsepower is equal to the real power sent to the motor. This is the electricity delivered to the motor. The motor is not rated based on this input. It is based on the output.

**The correct answer is most nearly, (b) 5 HP.**

- (a) 3 HP
- (b) 5 HP
- (c) 7.5 HP
- (d) 10 HP



Calculate the pressure due to the elevation difference.

$$H_{elev} = 75 \text{ ft head}$$

Next, calculate the vapor and velocity pressure at the suction of the pump.

$$H_{vapor} = 0.36 \text{ psi} * 2.307 \frac{\text{ft head}}{\text{psi}} = 0.83 \text{ ft head}$$

$$H_{velocity} = (2.52 \text{ ft/s})^2 * \frac{1}{2 * 32.2 \frac{\text{lbm-ft}}{\text{lbf-s}^2}} = 0.10 \text{ ft head}$$

Finally, add up all the pressures to the suction of the pump.

$$NPSHA = 28.2 + 75 - 1.96 + 0.1 - 0.83 = 100.4 \text{ ft head}$$

The correct answer is most nearly, **(c) 100 ft head**.

- (a) 25 ft head
- (b) 75 ft head
- (c) 100 ft head
- (d) 110 ft head

## 12.18 SOLUTION 18 – DISTRIBUTION SYSTEMS

Water is pumped from tank 1 to tank 2 in the figure below at a rate of 100 GPM. Both tanks are open to atmosphere. The system is made up of 20 ft of 3" schedule 40, welded steel piping with flanged fittings. Assume a roughness coefficient of 0.0018 inches, four 90° long radius elbows, two gate valves, one sharp entrance, and one sharp exit. What will be the pressure required at the pump?

Assume the following conditions:

Viscosity of water  $\nu = 1.08 \times 10^{-5} \text{ ft}^2/\text{s}$

Schedule 40 Steel Pipe Inside Diameter = 3.07 inches

Schedule 40 Steel Pipe Outside Diameter = 3.5 inches

Sharp Exit,  $K=1.0$

Sharp Entrance,  $K=0.5$

3" Flanged 90° long radius elbows,  $K=0.25$

3" Flanged 90° gate valve,  $K=0.22$



## 12.9 PROBLEM 9 – COOLING LOAD

A cooling coil has an operating temperature of 50 F and the manufacturer indicates that at 2,000 CFM and 80 F, the coil has a bypass factor of 5%. What is the leaving air temperature from the coil?

- (a) 50 °F
- (b) 52 °F
- (c) 78 °F
- (d) 80 °F

## 12.10 PROBLEM 10 - TURBINE

A turbine operates at an incoming pressure of 300 psia and an outgoing pressure of 5.0 psia. The incoming temperature of the steam is 1,000 F. If the turbine is 80% efficient, then what is the outgoing temperature of the steam?

- (a) 80 °F
- (b) 142 °F
- (c) 162 °F
- (d) 280 °F

$$s_{in} = s_{out} = 1.797$$

Using the final entropy, you can find the steam quality,  $x$ .

$$s_{out} = 1.797 = s_f + x * s_{fg} = 0.2348 + x * 1.6092$$

$$x = 0.97$$

Next, find the ideal enthalpy at the exit of the turbine.

$$\text{Steam tables} \rightarrow 5.0 \text{ psia}, x = 0.97 \rightarrow h_{out,ideal} = h_f + x * h_{fg}$$

$$h_{out,ideal} = 130.13 + x * 1000.57 = 1,100.7 \frac{\text{Btu}}{\text{lbm}}$$

The ideal amount of work produced by the turbine is as follows.

$$Q_{ideal} = \dot{m} * (h_{in} - h_{out,ideal}) = \dot{m} * (1,530 - 1,110.7)$$

But the turbine is 80% efficient, so only 80% of the ideal work is produced.

$$\text{Efficiency}_{turbine} = 0.8 = \frac{Q_{actual}}{Q_{ideal}} = \frac{Q_{actual}}{\dot{m} * 429.3} \rightarrow Q_{actual} = \dot{m} * 343$$

Next find, the actual enthalpy exiting the turbine.

$$Q_{actual} = \dot{m} * 343 = \dot{m} * (h_{in} - h_{out,actual})$$

$$\dot{m} * 343 = \dot{m} * (1,530 - h_{out,actual})$$

$$h_{out,actual} = 1,186.6 \frac{\text{Btu}}{\text{lbm}}$$

Since you now have two properties, enthalpy and pressure, you can find the exit temperature.

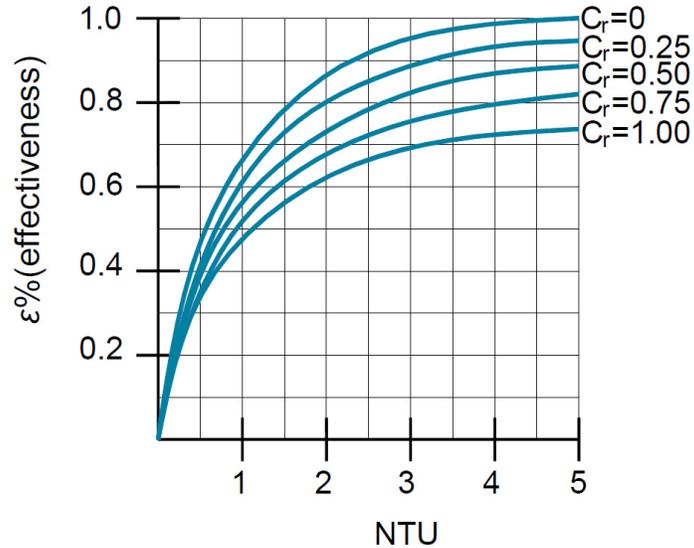
Navigate to your steam tables and you will find that at 5.0 psia, the enthalpy of saturated gas is 1,130.7 Btu/lbm and the enthalpy of saturated liquid is 130.2 Btu/lbm. Thus the actual enthalpy is in the superheated region, where the temperature is linearly related to enthalpy.

$$T_f = 280 \text{ F}$$



## 8.11 PROBLEM 11 - HEAT EXCHANGER

A counterflow, 3 pass plate and frame heat exchanger has an average  $U$  value of  $500 \text{ Btu/h-ft}^2\text{-F}$ .  $200 \text{ gpm}$  of water enters the heat exchanger at  $140 \text{ deg F}$  and  $100 \text{ gpm}$  of water enters the heat exchanger at  $70 \text{ deg F}$ . The area of heat transfer is  $250 \text{ ft}^2$ . What is the actual rate of heat transfer across the heat exchanger? **The correction factor is unknown.**



- (a) 2.7 MMBtuh
- (b) 3.5 MMBtuh
- (c) 5.6 MMBtuh
- (d) 8.7 MMBtuh

## 5.5 PROBLEM 5 – ENERGY RECOVERY

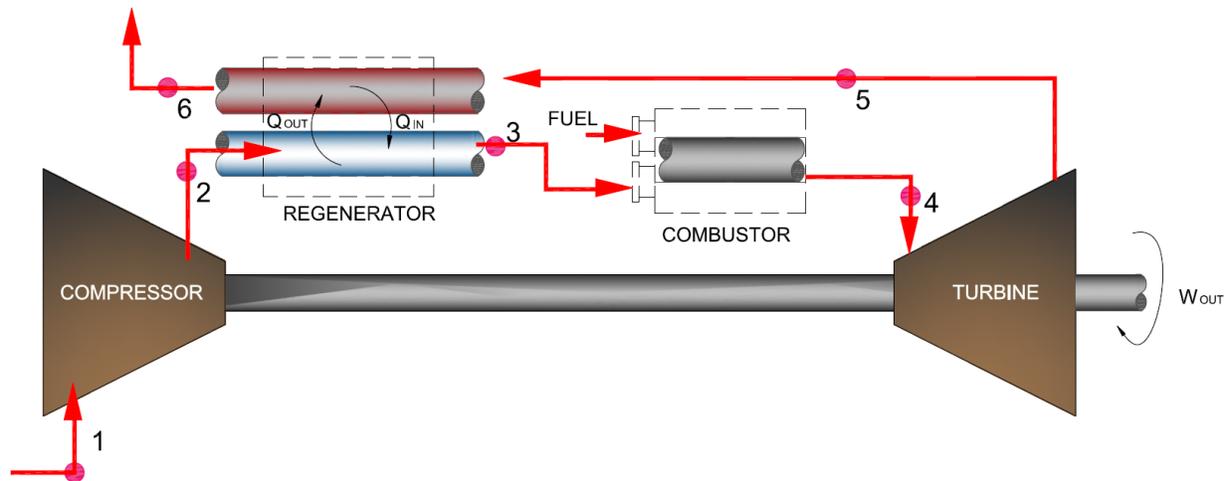
An underground cavern is used to store compressed air at a pressure of 300 psia. The compressor's incoming conditions are 80 °F at 14.7 psia. The air mass flow rate is 50,000 lbm/hr. The compressor has an isentropic efficiency of 75%. The compressor is connected to a motor which has an efficiency of 90%. If the compressor is run for 4 hours a day for 365 days a year, then what is the required electricity?

- (a) 2,823 MWH per year
- (b) 3,050 MWH per year
- (c) 4,182 MWH per year
- (d) 5,577 MWH per year

## 5.6 PROBLEM 6 – ENERGY RECOVERY

A gas turbine engine intakes air at 90.3°F, 14.7 psia. Assume a constant air specific heat of 0.245 Btu/(lbm-°F) and  $k = 1.4$ . The compressor pressure ratio is 6.5. The air temperature leaving the combustor is 3,000°F. Assume no pressure drop through the regenerator and combustor and a constant mass flow rate at all points in the cycle. The compressor has an efficiency of 70%. What is the thermal efficiency of the cycle?

Point 3:  $T = 1,500$  °F; Point 5:  $T = 2,000$  °F.



- (a) 30%
- (b) 34%
- (c) 39%
- (d) 44%

$$80\text{ }^\circ\text{F DB}, 14.7\text{ psia} \rightarrow P_{1,r} = 1.386; h_1 = 129 \frac{\text{Btu}}{\text{lb}}$$

$$P_{2,r} = 1.386 * \left(\frac{300}{14.7}\right) = 28.29$$

$$h_{2,ideal} = 305 \frac{\text{Btu}}{\text{lb}}$$

Next, find the compressor work by multiplying the mass flow rate by the change in enthalpy. The compressor will input more work than the ideal amount due to the 75% efficiency value.

$$P = 50,000 \frac{\text{lb}}{\text{hr}} * \left(305 - 129 \frac{\text{Btu}}{\text{lb}}\right) * \left(\frac{1}{.75}\right) = 11,733,333 \frac{\text{Btu}}{\text{h}}$$

Next, convert the compressor power to kW.

$$P_{compressor} = 11,733,333 \frac{\text{Btu}}{\text{h}} * \left(\frac{1 \text{ kW}}{3,413 \frac{\text{Btu}}{\text{h}}}\right) = 3,438 \text{ kW}$$

The motor will convert electricity into power for the compressor. The motor will have some inefficiency losses, so the electricity input to the compressor will be slightly higher than the compressor power.

$$P_{motor} = \frac{3,438 \text{ kW}}{0.9} = 3,820 \text{ kW}$$

Lastly, multiply kW by the operational hours.

$$3,820 \text{ kW} * \left(\frac{4 \text{ h}}{\text{day}}\right) \left(\frac{365 \text{ days}}{1 \text{ yr}}\right) = 5,576,933 \text{ kWh}$$

The correct answer is most nearly, (d) **5,577 MWH** per year.

- (a) 2,823 MWH per year
- (b) 3,050 MWH per year
- (c) 4,182 MWH per year
- (d) 5,577 MWH per year

## 6.6 SOLUTION 6 - ENERGY RECOVERY

A gas turbine engine intakes air at 90.3°F, 14.7 psia. Assume a constant air specific heat of 0.245 Btu/ (lbm-°F) and k = 1.4. The compressor pressure ratio is 6.5. The air temperature

The correct answer is most nearly, (d) 280 °F.

- (a) 80 °F
- (b) 142 °F
- (c) 162 °F
- (d) 280 °F

### 13.11 SOLUTION 11 – BRAYTON CYCLE

Air enters a compressor at atmospheric pressure and a temperature of 70 F. The compressor has a pressure ratio of 8 and the combustion exit temperature is 1,500 F. What is the back work ratio?

The back work ratio is the ratio of the work produced by the turbine that is used to power the compressor.

$$\text{Back work ratio} = \frac{Work_{comp}}{Work_{turb}};$$

The work for the compressor and turbine can be found with the change in temperature between the entering and exit conditions of each piece of equipment.

$$Work_{comp} = \dot{m} * c_p * (T_{out,comp} - T_{in,comp})$$

$$Work_{turb} = \dot{m} * c_p * (T_{in,turb} - T_{out,turb})$$

First the compressor entering temperature is 70 F and in absolute temperature, 530 R.

The exit temperature from the compressor can be found with the isentropic relationship below.

$$T_{out,comp} = T_{in,comp} \left( \frac{P_{out,comp}}{P_{in,comp}} \right)^{\frac{k-1}{k}}$$

$$T_{out,comp} = 530 \text{ °R} * (8)^{\frac{1.4-1}{1.4}} = 960 \text{ °R}$$

Next the turbine exit condition can be found with the same relationship but in reverse, since it is isentropic expansion. The entering turbine temperature is the maximum temperature of 1,500 F or 1,960 R.

$$T_{out,turb} = T_{in,turb} \left( \frac{P_{out,turb}}{P_{in,turb}} \right)^{\frac{k-1}{k}}$$

$$T_{out,turb} = 1,960 \left( \frac{1}{8} \right)^{\frac{1.4-1}{1.4}} = 1,082 \text{ °R}$$

### 5.3 PROBLEM 3 – COMBINED CYCLES

A combined cycle gas-steam turbine has a steam turbine efficiency of 90%, a gas turbine efficiency of 87% and a compressor isentropic efficiency of 80%. What is the air outlet temperature from the compressor?

The air conditions entering the compressor are 70 °F, 14.7 psia. The air leaving the compressor is at 200 psia. The air leaving the combustor is at 2,500 °F, 200 psia. The air leaving the turbine is at 25 psia. The air leaving the heat recovery/steam generator is at 250 °F. The steam entering the heat recovery/steam generator is at 1,000 psia, 110 °F. The steam leaving the heat recovery/steam generator is at 1,000 psia, 1,000 °F. The steam leaving the steam turbine is at 5 psia. Assume  $k_{\text{air}} = 1.4$ .

- (a) 555 °F
- (b) 598 °F
- (c) 657 °F
- (d) 804 °F

### 5.4 PROBLEM 4 – COMBINED CYCLES

A combined cycle gas-steam turbine has a steam turbine, isentropic efficiency of 90%, a gas turbine efficiency of 87% and a compressor isentropic efficiency of 80%. What is the steam turbine output work?

The air conditions entering the compressor are 70 °F, 14.7 psia. The air leaving the compressor is at 200 psia. The air leaving the combustor is at 2,500 °F, 200 psia. The air leaving the turbine is at 25 psia. The air leaving the heat recovery/steam generator is at 250 °F. The steam entering the heat recovery/steam generator is at 1,000 psia, 110 °F. The steam leaving the heat recovery/steam generator is at 1,000 psia, 1,000 °F. The steam leaving the steam turbine is at 5 psia. Assume  $k_{\text{air}} = 1.4$ .

- (a) 445 Btu/lb
- (b) 494 Btu/lb
- (c) 549 Btu/lb
- (d) 633 Btu/lb

Lastly, the turbine efficiency can be used to find the actual leaving temperature or actual leaving enthalpy. Please remember that the turbine will actually output less due to the inefficiencies, therefore the actual output will be the smaller number and should be in the numerator.

$$\text{Turbine Efficiency} = \frac{\text{Actual Output Work}}{\text{Ideal Work}}$$

$$90\% = \frac{\text{Actual Output Work}}{494 \frac{\text{Btu}}{\text{lb}}}$$

$$\text{Actual Output Work} = 445 \frac{\text{Btu}}{\text{lb}}$$

The correct answer is most nearly, (a) **445 Btu/lb**.

- (a) 445 Btu/lb
- (b) 494 Btu/lb
- (c) 549 Btu/lb
- (d) 633 Btu/lb

## 6.5 SOLUTION 5 - COMBINED CYCLES

A combined cycle gas-steam turbine has a gas turbine with an input fuel HHV of 21,000 Btu/lb and a combustor efficiency of 85%. What is the require fuel mass flow rate?

The air conditions entering the compressor are 70 °F, 14.7 psia. The air leaving the compressor is at 200 psia. The air leaving the combustor is at 2,500 °F, 200 psia. The air leaving the turbine is at 25 psia. The air leaving the heat recovery/steam generator is at 250 °F. The steam entering the heat recovery/steam generator is at 1,000 psia, 110 °F. The steam leaving the heat recovery/steam generator is at 1,000 psia, 1,000 °F. The steam leaving the steam turbine is at 5 psia. Assume  $k_{\text{air}} = 1.4$  and  $c_{p,\text{air}} = 0.27 \text{ Btu/lb-}^\circ\text{F}$ .

Next, create an energy balance at the combustor between the heat gained by the air and the heat released by the fuel.

$$\text{Efficiency} = \frac{\text{Output Change in Compressed Air}}{\text{Input Fuel}}$$

$$85\% = \frac{\dot{m}_{\text{air}} * c_p * (T_{\text{out}} - T_{\text{in}})}{\left(\dot{m}_{\text{fuel}} \frac{\text{lb}}{\text{hr}}\right) * (\text{HHV} \frac{\text{Btu}}{\text{lbm}})}$$

$$85\% = \frac{\dot{m}_{\text{air}} * 0.27 \frac{\text{Btu}}{\text{lb-F}} (2500 - 580)}{\left(\dot{m}_{\text{fuel}} \frac{\text{lb}}{\text{hr}}\right) * (21,000 \frac{\text{Btu}}{\text{lbm}})}$$